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Abstract

This paper studies whether relaxing the noise trader assumption in Kyle (1985) results in an equilibrium solution which is equivalent to the original solution in the following sense: market depth, the expected total trading volume and the expected price level have the same equilibrium values as in the original model. Conditions are derived for which the "hedging" model of Spiegel and Subrahmanyam (1992) is shown to be equivalent, both when the number of uninformed traders is finite as well as when it goes to infinity in a particular way. The equilibrium solution of a version of Glosten (1989) is only partially equivalent, but any deviation in the equilibrium values from those in Kyle (1985) is bounded.

ON THE ROBUSTNESS OF MARKET MICROSTRUCTURE MODELS

In his remarks introducing a market microstructure symposium, Chester Spatt (1991) attested to the need for assessing "the robustness of microstructure analyses," in particular by considering the "trading incentives of 'liquidity' traders." While there have been several recent advances in this direction, most of these papers have not specifically addressed the question of robustness. A notable exception is Spiegel and Subrahmanyam (1992) who extend the framework of Kyle (1985) by modeling the noise traders as risk-averse investors who trade to "hedge" their endowment of shares of the risky asset. The focus of their paper is on showing that several comparative static results in Kyle (1985) are upset by their generalization and they conclude by stating that "postulating the existence of noise traders with exogenous demands therefore, is not solely a convenient modeling device . . . it is an assumption whose relaxation alters many of the basic results."

This paper takes a different, though complementary, tack. Instead of considering comparative static results, it looks at the equilibrium solution of the Kyle (1985) model[1] and asks the following question. Consider an alternative equilibrium solution that results when the noise trader assumption is relaxed. When are these two solutions equivalent in the following sense: the equilibrium values of market depth, the expected total trading volumes and the expected price levels are exactly the same?[2] Notice that "equivalence" is defined with respect to aggregate market variables,[3] which a casual market observer might reasonably be expected to know or find out.[4] If there are two markets which are equivalent in the above sense, such an observer should, on average, be indifferent as to which market he would like to trade in. In other words, the apparently "separate" models are really describing the same underlying market.

Specifically, in this paper I compare the equilibrium solution in Kyle (1985) with those in two alternative representations of uninformed trading: the hedging model of Spiegel and Subrahmanyam (1992) and a version of Glosten (1989).[5] The common element in all three models is that price determination occurs through the batch market mechanism. The former model can be viewed as a model of *uninformed traders* (i.e., there is a distinct group of investors whose trading motives are unrelated to information) whereas the latter is a model of *uninformed trading*, in that the same investor trades for both risk-sharing and information reasons. It turns out that this distinction is crucial. In particular, it is shown that, while equivalence exists between Kyle (1985) and the hedging model, the expected price level is strictly lower in Glosten (1989) relative to its equilibrium value in Kyle (1985), ruling out full equivalence between the latter two models. The reason for this is explained below.

In Glosten (1989) there is a single risk-averse investor who trades for both risk-sharing as well as for informational reasons. Let $1/\Gamma$ denote the market depth. Then the investor's trades are, in equilibrium, inversely proportional to the sum of Γ plus a risk-premium, unlike the other two models where an individual risk-neutral informed trader's equilibrium trades are proportional to the market depth only. This implies that, to sustain the same volume of trading as in the other two models, market depth must be higher or Γ lower in Glosten (1989). Conversely, to maintain the same market depth, the equilibrium total trading volume must be lower. Hence, the expected price level (which is a product of Γ and the expected total trading volume) must be lower as well.

For the hedging model, equivalence is derived for two separate cases. When the number of hedgers is finite, the hedgers' risk-aversion parameter must have a specific value which depends upon the variances of both the asset value and the hedgers' endowments as well as the amount of information in the market. Of course, the degree of risk-aversion is an exogenous parameter and so there is nothing to ensure that this value will in fact be attained. Thus, equivalence is also derived for a limit economy where the equivalence condition is independent of the value of the risk-aversion parameter. Let k be the number of uninformed investors each with a random endowment w of shares of a risky asset and let Σ_w be the variance of this endowment. Then, equivalence holds if k tends to infinity in such a way that the total amount of noise $k\Sigma_w$ tends to a finite number (which implicitly requires that $\Sigma_w \to 0$ as $k \to \infty$). This result formalizes the notion that the equilibrium outcome of Kyle (1985) can be viewed as the limiting solution to a game with one or more risk-neutral informed traders and a large number of risk-averse and atomistic uninformed traders who trade for risk-sharing reasons alone.

For the Glosten (1989) model, although full equivalence is impossible, partial equivalence exists since the equilibrium values of either the market depth or the expected total trading volume can separately (but not simultaneously) be equal to their corresponding values in Kyle (1985). In addition, the differences in the equilibrium values of the variables between the two models is bounded from above. Finally, under a further restriction, the equilibrium market depth has the same value in all three models and the (unconditional) expected utility of Glosten's investor is equal to the (unconditional) expected profits of a single informed trader in the other two models.

Overall, given this paper's particular approach to the robustness question, Kyle's (1985) model turns out to be surprisingly robust with respect to a relaxation of the noise trader assumption.[6] For the class of uninformed *trader* models, it is possible to move in a continuous fashion between Kyle (1985) and these models simply by changing the number of uninformed traders while keeping the total noise in the market fixed. For the class of uninformed *trading* models, the lack of full equivalence can be traced to the additional dichotomy created due to the lack of a group of risk-neutral purely informed traders in these models. Even in this case, however, it is possible to show that the equilibrium solutions cannot be arbitrarily far apart. Thus, while the importance of examining different models which deviate from the noise trader assumption remains, the researcher can also take comfort from observing a deeper unity connecting the original paradigm of Kyle (1985) to some of the successor models.

The two representations of uninformed trading studied in this paper were chosen for their ease of comparability to the Kyle (1985) model. Other valid models of uninformed trading exist as well. One of the earliest attempts at modeling uninformed traders as strategic players was in Admati and Pfleiderer (1988) who allow uninformed traders to choose when to trade (although not how much). Similar approaches can be found in Chowdhry and Nanda (1991) and Subrahmanyam (1991). Laffont and Maskin (1990) study a model with a risk-neutral price-setting insider and a continuum of risk-averse uninformed traders. They study both separating and pooling equilibria and derive conditions such that all investors would prefer the pooling equilibrium. In Bhattacharya and Spiegel (1991) both the monopolist insider and the continuum of uninformed traders are risk-averse. Prices are set through a Walrasian auction mechanism and both linear and non-linear trading strategies are derived.

An interesting approach to the uninformed traders issue is found in two papers by Gorton and Pennacchi (1990, 1992). In their models, all investors have endowments of a consumption and a capital good and live for three periods. Some subset of uninformed investors must consume early and so are forced to trade with informed traders. However, uninformed traders may contract with financial intermediaries or trade in index-linked securities to protect themselves from exploitation by informed traders. The intermediaries protect uninformed traders by splitting the cash flows into equity and (relatively) riskless debt, with informed traders holding the former and uninformed traders the latter. Index-linked securities have a lower return variance relative to the underlying primitive securities and thus reduce the expected trading losses of the uninformed traders.

The paper is organized as follows. Section I lays out a generalized version of Kyle (1985) and restates the results of Spiegel and Subrahmanyam (1992). Section II shows the equivalence of these two models for both the finite and the limit economy. Section III derives a version of Glosten (1989) and compares its equilibrium solution to that of Kyle (1985). Section IV concludes. All proofs are in the Appendix.

I. Uninformed Traders as Hedgers

In Section A, a generalized version of the model of Kyle (1985) is described with multiple informed traders and imperfect information. This is followed in Section B with a statement of the model of Spiegel and Subrahmanyam (1992).

A. The Noise Trading Model

Consider a market in which a single risky asset with an unknown liquidation value v is traded. There is a group of m informed traders each of whom receive, prior to trading, signals s^i about the unknown value v. The signals are of the form $s^i = v + e^i$, $i=1,\ldots,m$ where the error terms e^i are independent of each other. In addition, there is a group of uninformed noise traders who trade for liquidity reasons. The uninformed traders' motives for trading are not modeled.

Each informed trade $i=1,\ldots,m$ submits a market order x^i to a market maker. The noise traders also collectively submit market orders worth u to the same market maker for execution. The latter then fixes a single price p_n at which she will execute the total order flow $y_n = x_n + u$, where x_n is the combined market orders of the informed group in the noise trader model. The market maker is assumed to be risk-neutral and competitive. Conditional on observing y_n, she earns zero expected profits.

The random variables in the model are v, u and e^i, i=1,…,m. All these variables are normally distributed with zero mean and finite variances Σ_v, Σ_u and Σ_e, respectively. Thus the m error terms are drawn from an identical distribution. In addition, all informed investors follow linear trading rules $x^i = A_n s^i$, i=1,…,m. This implies that the market maker's pricing rule is also linear: $p(y_n) = \Gamma_n y_n$, where $\Gamma_n = Cov(v,y_n)/Var(y_n)$ is the now-familiar market depth parameter.

Define $t = \Sigma_v/(\Sigma_v+\Sigma_e)$ and note that $0 \le t \le 1$. t is a measure of the unconditional precision of s^i, i=1,…,m. For example, if t = 1 then s^i is a perfect signal. Further, define $Q = [1+t(m-1)]$ where 1/Q is a measure of the posterior valuation of v conditional on observing the m-vector of signals $(s^1,\dots,s^m)$. Finally, let s denote the sum of all the signals, i.e., $s = \sum_{i=1}^{m} s^i$. Lemma 1 describes the equilibrium solution for the noise trader model and Lemma 2 describes its properties. (In Lemma 2, price informativeness $PI_n = \Sigma_v - Var(v|p_n)$.)

Lemma 1: In the noise trader model, each informed trader i=1,…,m trades $x^i = A_n s^i$ and the price is $p_n = \Gamma_n y_n$, where $A_n = \frac{t}{\Gamma_n(1+Q)}$ and $\Gamma_n = \frac{\sqrt{mt\,\Sigma_v}}{(1+Q)\sqrt{\Sigma_u}}$.

Lemma 2: The noise trader model is characterized by the following properties:

1. **Market depth $1/\Gamma_n$ is proportional to the ratio of noise to information in the market.**
2. **Price informativeness $PI_n = \frac{mt\,\Sigma_v}{1+Q}$ and is independent of the amount of noise.**

3. **Total (unconditional) expected profits of the informed $I_n = \frac{\sqrt{mt\,\Sigma_v \Sigma_u}}{1+Q}$ and is proportional to the standard deviations of noise and asset value and the square root of the total amount of information mt in the market.**

4. **Total (conditional) expected profits of the informed $C_n = \frac{t^2 s^2}{\Gamma_n Q (1+Q)^2}$.**

Next, the basic model is extended to allow for rational behavior by uninformed traders.

B. The Model With Uninformed Traders as Rational Hedgers

There are k risk-averse uninformed traders ("hedgers") who trade for purely risk-sharing reasons. The development of the model here follows Spiegel and Subrahmanyam (1992). Each hedger j has random endowment w^j, which is assumed to be normally distributed with mean zero and variance Σ_w. w^j, j=1,…,k are independent of each other and all other random variables in the model. All hedgers have negative exponential utility functions with risk-aversion parameter R_h.

Suppose that all hedgers submit market orders u^j to the market maker and follow linear trading rules of the form $u^j = Dw^j$, j=1,…,k. Let the total uninformed trades be $u_h = \sum_{j=1}^{k} u^j$. If π_h^j is the profit of the j-th hedger, then u^j is chosen to maximize her utility or certainty-equivalent profits $V_h^j = E\left(\pi_h^j | w^j\right) - \frac{R_h}{2} \mathrm{Var}\left(\pi_h^j | w^j\right)$. The informed traders maximization problem remains the same as before, since each w^j is independent of v.[7]

Market depth is now positively related to the magnitude of the "hedge factor" D (since this increases the variance of the total order flow) and to the risk aversion parameter R_h. Further, the equilibrium $D < 0$ since the marginal utility of the hedgers from a purchase (sale) is negative if endowments are positive (negative).

Let x_h and Γ_h be the total informed trading volume and the market depth parameter in the hedger model. Lemma 3 (which is identical to Proposition 1 in Spiegel and Subrahmanyam (1992)) describes the equilibrium. Equilibrium exists if the amount of risk-aversion and noise in the market exceeds the amount of information available. Lemma 4 is a comparison of the equilibrium properties of the current model with that of Kyle (1985). The comparison is expressed by relating the equilibrium values of the relevant variables in these two models (part 2 of the lemma is contained in Spiegel and Subrahmanyam (1992)). $|D|$ refers to the absolute value of D.

Lemma 3: An equilibrium to the hedger model exists if R_h satisfies:

$$N = R_h(2-t)\sqrt{k\Sigma_w\Sigma_v} - 2\sqrt{mt} > 0. \qquad (1)$$

In equilibrium, each hedger j=1,...,k trades $u^j = Dw^j$, where $D < 0$, market depth is $1/\Gamma_h$ and:

$$\Gamma_h D = -\frac{\sqrt{mt\,\Sigma_v}}{(1+Q)\sqrt{k\Sigma_w}} \qquad (2)$$

$$D = \frac{-N(1+Q)k}{R_h\Sigma_v[(2-t)k(1+Q)-mt]} \qquad (3)$$

Each informed trader $i=1,\dots,m$ trades $x^i = A_h s^i$, where $A_h = \frac{t}{\Gamma_h(1+Q)}$. The price is $p_h = \Gamma_h y_h$, where $y_h = x_h + u_h$.

Lemma 4: The hedger model is characterized by the following properties:

1. **Market depth $\frac{1}{\Gamma_h} = \frac{-D\sqrt{k\Sigma_w}}{\Gamma_n\sqrt{\Sigma_u}}$.**
2. **Price informativeness $PI_h = PI_n$.**
3. **Unconditional expected profits of the informed group $I_h = I_n \frac{|D|\sqrt{k\Sigma_w}}{\sqrt{\Sigma_u}}$.**
4. **Conditional expected profits of the informed group $C_h = C_n \frac{\Gamma_n}{\Gamma_h}$.**

Comparing Lemmas 2 and 4, it is apparent that the two models of uninformed trading differ essentially along two dimensions: the magnitude of the hedge factor D and the amount of noise which is Σ_u in one case and $k\Sigma_w$ in the other. These two factors, in turn, determine the differences in equilibrium market depth, informed profits and trading volumes.

II. The Equivalence of the Two Models

In this section, it is shown that there exist conditions under which the equilibrium solutions of the two models are equivalent. I will first define what is meant by "equivalence of the two models."

Definition 1: The two models of uninformed trading described in Section I will be considered equivalent if the equilibrium values of the following variables are equal: market depth, the expected total trading volumes[8] and the expected price levels.

Even if Definition 1 is satisfied, the two models share the same equilibrium solution "on average" only since it is the expected and not the actual total trading volumes (and, by implication, the actual prices) which are being equated. There is no sensible way that the actual total trading volumes can be equated since this would require the actual uninformed trading volumes to be equated.

Corollary 1: If the conditions in Definition 1 are met, then the following variables are also equal in the two models: the actual and expected informed trading volumes and profits and the expected uninformed trading volumes.

Thus, Corollary 1 implies that an implication of equivalence is that the equilibria in the two models would be characterized in identical fashion (refer to Lemmas 2 and 4). It is easy to show that price informativeness PI is directly proportional to the square of the expected price level. Thus $PI_h = PI_n$ implies that the expected price levels are also equal. The conditions under which the equilibrium values of the other two variables named in Definition 1 would be equated are discussed next.

A. Equivalence in a Finite Economy

Suppose that the amount of noise in the two economies are pegged at the same level, i.e., $\Sigma_u = k\Sigma_w$. Then, from the discussion following Lemma 4, if D = -1 (uninformed traders hedge perfectly), the two models are equivalent in the sense of Definition 1. The condition for which D = -1 is satisfied requires that the risk-aversion parameter R_h satisfy a particular value given in Proposition 1. It can be shown that, at this value, R_h is directly proportional to the equilibrium market depth.

Proposition 1: Suppose that $\Sigma_u = k\Sigma_w$. Then if $R_h = \frac{2(1+Q)\sqrt{k}}{\sqrt{mt\Sigma_v\Sigma_w}}$, the two models of uninformed trading are equivalent.

Given the value of R_h specified in Proposition 1, D = -1 and each uninformed trader hedges all of his endowment in equilibrium. Uninformed traders as a group trade (the negative of) $w_h = \sum_{j=1}^{k} w^j$. Thus, the equilibrium behavior of the hedgers mimic that of the noise traders. The additional assumption $k\Sigma_w = \Sigma_u$ amounts to a scaling of these variances ensuring that the total amount of noise in the two models is the same.

In an economy with a finite number of hedgers, the risk aversion parameter R_h is required to have a specific value. Since R_h is exogenous to the model, a more natural outcome is one where the equivalence holds for *any* value of R_h. Such an outcome is described in the next sub-section.

B. Equivalence in a Limit Economy

In the limit, as the number of hedgers k goes to infinity, D goes to -1. However, without further restrictions on the parameters, equilibrium fails to exist in the limit as the market becomes infinitely deep ($\Gamma_h \to 0$). To ensure the existence of an equilibrium, I make the following assumption:

Assumption 1: As $k \to \infty$, $k\Sigma_w \to Z > 0$ (Z finite).

For Assumption 1 to hold, it must be true that as the number of hedgers k increases, the variance of each hedger's endowment becomes smaller ($\Sigma_w \to 0$) in such a way that the total amount of noise $k\Sigma_w$

remains a constant. The effect of this assumption is to assure that, in the limit economy, each uninformed trader is atomistic and the group of uninformed traders behaves as a continuum.

Proposition 2: Suppose Assumption 1 holds. If $Z = \Sigma_u$, then the two models of uninformed trading described in Section I are equivalent.

The additional assumption $Z = \Sigma_u$ is analogous to the assumption $k\Sigma_w = \Sigma_u$ in Proposition 1 and serves the same purpose. Proposition 2 imposes less restrictive conditions for the equivalence to be satisfied, in that it does not require any specific value of the risk-aversion parameter R_h. Thus, the model of Kyle (1985) can be viewed as the limiting case of a larger game where the noise traders are fully rational agents. This continuity property is important because it ensures that there is no sudden "jump" in the equilibrium outcome in moving from the larger game to its limiting solution.

III. A Model of Uninformed Trading

Both the models discussed in Section II emphasize uninformed traders rather than uninformed trading. In other words, a distinct group of traders are postulated who engage exclusively in trading activities unrelated to information. An alternative approach, found in Glosten (1989) (and following the earlier work of Glosten and Milgrom (1985)), considers an individual trader who sometimes engages in information-related and sometimes in risk-sharing trading activities.

The market in Glosten (1989) follows the microstructure of the market in Glosten and Milgrom (1985). In this market, the market maker posts a pricing schedule $p(y_g)$ which specifies the price that a trader will obtain corresponding to any quantity y_g he demands. To facilitate the comparison with Kyle (1985), I will substitute the batch market mechanism for this posted-price mechanism while keeping all the other features of the original model intact. Further, I will only focus on the model with competitive market makers. Equivalence will again be as described in Definition 1.[9]

It will be shown that there are no set of conditions such that these two models are equivalent in the sense of Definition 1. While either market depth or the expected total trading volume (but not both) can have the same equilibrium values in the two models, the expected price level is always strictly lower in the Glosten (1989) model.

There is a single risk-averse investor with private information s about the value of the single risky asset v, where $s = v + e$ and e is uncorrelated with v. The investor is also endowed with w shares of the risky asset. v, e and w are normally distributed with zero mean and variances Σ_v, Σ_e and Σ_w. The investor has negative exponential utility with risk-aversion parameter $R_g > 0$.

The investor submits his market order y_g to a single risk-neutral and competitive market maker who sets a price p_g at which the order will be executed. In the usual way, $p_g = E(v|y_g)$. The market maker is unaware when the investor's trades are information-based and when they are not. He conjectures that $y_g = B_1 s + B_2 w$. This leads to a linear price schedule $p_g = \Gamma_g y_g$. Let t be as defined in Section 1, i.e., $t = \Sigma_v/(\Sigma_v+\Sigma_e)$. Then Lemma 5 describes the equilibrium of this model.

Lemma 5: Suppose $0 < t < 1$. Define $G = R_g^2(1-t)^2\Sigma_v\Sigma_w$. If $G > t$, then there is a unique equilibrium to the trading model in which the investor trades $y_g = B_1 s + B_2 w$ and the price is $p_g = \Gamma_g y_g$ where:

$$B_1 = \frac{t\sqrt{\Sigma_v}}{2\Gamma_g\sqrt{\Sigma_v} + \sqrt{G\Sigma_v}}, \quad B_2 = \frac{-\sqrt{G\Sigma_v}}{2\Gamma_g\sqrt{\Sigma_v} + \sqrt{G\Sigma_v}}, \quad \text{and} \quad \Gamma_g = \frac{t\sqrt{G\Sigma_v}}{\sqrt{\Sigma_v}(G-t)}. \tag{4}$$

Remember that t is a measure of the unconditional precision of the information signal s. The equilibrium of Lemma 5 says that t also measures the weight that the investor puts on each aspect of his trading activities. The more precise is his information, the larger the weight placed on his information-related trading activities. In the extreme, when $t = 1$, the investor optimally ignores his endowment of w (i.e., $B_2 = 0$) and acts as a pure informed trader. Of course, with no noise trading, this cannot be an equilibrium. Similarly, when $t = 0$, the investor is exclusively concerned with risk-sharing, $B_1 = 0$ and again equilibrium does not exist.

As in Spiegel and Subrahmanyam (1992), the product of risk-aversion and noise (named G here) must be sufficiently large for equilibrium to exist and further, the equilibrium hedge factor $B_2 < 0$. Unlike Spiegel and Subrahmanyam (1992), the magnitude of $B_2 < 1$. Uninformed traders are never perfect hedgers nor do they ever "overhedge." The equilibrium market depth is positively related to risk-aversion R_g and negatively related to the information precision t.

Next, consider the relationship between this model and that of Kyle (1985). Without loss of generality, the comparison made will be with the solution of the Kyle model when the number of informed traders $m = 1$. Proposition 3 shows that either Γ or the expected total trading volume can separately have the same equilibrium values in the two models. But if the value of one of these variables is equalized, the equilibrium value of the other variable is strictly lower in Glosten by a common factor of proportionality $F < 1$. Further, the expected price level is also strictly lower in Glosten (1989) by the same factor of proportionality F.

Proposition 3: Define $F = \frac{\sqrt{2t}}{\sqrt{G+t}} < 1$ and $k = \frac{\Sigma_u}{\Sigma_v}$. (1) If $\sqrt{k} = \frac{G-t}{2\sqrt{tG}}$, then $\Gamma_g = \Gamma_n$ and $E(|y_g|) = FE(|y_n|) < E(|y_n|)$. (2) If $\sqrt{k} = \frac{G-t}{\sqrt{2G}\sqrt{G+t}}$, then $E(|y_g|) = E(|y_n|)$ and $\Gamma_g = F\Gamma_n < \Gamma_n$. (3) $E(|p_g|) = FE(|p_n|) < E(|p_n|)$.

Proposition 3 essentially says that, to sustain the same level of trading as in the Kyle (1985) market, market depth must be higher in the Glosten (1989) model. Or to sustain the same depth of market, the level of trading needs to be lower. The reason for this is that the investor is risk-averse in Glosten (1989) and so B_1, which measures the intensity with which information is exploited, is inversely proportional to the *sum* of Γ_g plus a risk premium (see Lemma 5). Whereas the informed trader is risk-neutral in the other two models, so that the equilibrium trading intensity is proportional to market depth *only*. Clearly, if the same trading volume is to be sustained in all the models and $R_g > 0$, the market depth has to be higher in the Glosten (1989) version. Hence, the expected price level (which is a product of Γ and the expected total trading volume) must be lower as well.

Notice that $F < 1$ follows from the existence condition $G > t$ in Lemma 5. Therefore, Proposition 3 directly implies that equivalence in the sense of Definition 1 is not possible between the models of Kyle (1985) and Glosten (1989) because this requires the risk-aversion parameter R_g to be too low for equilibrium to exist.

Corollary 2: The models of Kyle (1985) and the version of Glosten (1989) defined in this section can never be equivalent in the sense of Definition 1.

From Proposition 3, the metric $1 - F$ can be interpreted as a measure of how much the equilibrium values of the variables differ between the two models. For example, $F = 1$ implies full equivalence. The next result shows that there exists $B(k)$, which is a function of k only, such that $(1-F) = B(k)$. $B(k)$ belongs to the open interval $(0,1)$ and is strictly increasing in k (which, it may be remembered, scales the noise level in the two models). Thus, the between-model differences in the equilibrium values cannot be arbitrarily far apart and, by choosing lower values of k, the differences can be made smaller. For example, from the expression for $B(k)$ given in equations (A20) and (A24) in the Appendix, $B(1) = 0.46$ while $B(8) = 0.75$.

Proposition 4: There exists a function $B(k)$, where $0 < B(k) < 1$, such that $(1-F) = B(k)$. $B(k)$ is strictly increasing in k and is derived in equations (A20) and (A24) of the Appendix.

The condition that ensures the equality of market depths in part 1 of Proposition 3 appears to be very different from the restriction on R_h

specified in Proposition 1. However, if the value of R_g implied by Proposition 3 is solved for, the following expression is obtained (this expression is derived in equations (A22) and (A23) of the Appendix):

$$R_g = \frac{1}{\sqrt{\Sigma_v \Sigma_w}} \frac{\sqrt{t}}{1-t} \left(\sqrt{k} + \sqrt{k+1}\right) = R_g^*. \tag{5}$$

For easy reference, I restate (after fixing the number of informed traders m = 1) the analogous condition which ensures the equality of the market depths between the models of Kyle (1985) and Spiegel and Subrahmanyam (1992) from Proposition 1:

$$R_h = \frac{1}{\sqrt{\Sigma_v \Sigma_w}} \frac{4\sqrt{k}}{\sqrt{t}} = R_h^*. \tag{6}$$

In fact, the similarity of (5) to (6) is quite striking. In both cases, the risk-aversion parameter is inversely proportional to the variances of the asset value and the hedger's endowment and directly proportional to k. It is possible to specify values of the information precision t and k such that $R_g^* = R_h^*$. Under these conditions, market depth is equalized across all three models.

Corollary 3: $R_g^* = R_h^*$ **if** $k = T^2/(16-8T)$ **where** $T = t/(1-t)$.

Numerical calculations show that for $k \geq 1$, only four values of t satisfy Corollary 3. These are shown in Table 1. One example is $k = 3.02$ and $t = 0.65$. Higher values of t must be supported by higher values of k.

In Section I, certain important properties of the equilibrium solution in the Kyle (1985) model had been identified in Lemma 2. The equivalence results in Section II simultaneously ensured that these properties were satisfied in the model of Spiegel and Subrahmanyam (1992) as well. Since full equivalence has been ruled out in this case, it is clear that some or all of these properties will not hold here. Corollary 4 states that price informativeness is always lower in the model of Glosten (1989) but that, if the market depths are equal, then the expected utility of the investor equals in magnitude the unconditional expected profits of Kyle's monopolist insider. PI_g denotes the informativeness of prices and V_g the certainty-equivalent profits of the investor in the Glosten (1989) model.

Corollary 4: (1) $PI_g = F^2 PI_n < PI_n$. (2) Suppose $\Gamma_g = \Gamma_n$. If $k = 1/3$, then $I_n = |E(V_g)|$.

Part (1) of the corollary follows directly from Proposition 3 if we note that price informativeness $PI_i = \Gamma_i^2 \Sigma_{y_i}$ for $i = g, n$ where Σ_{y_i} is the variance of the total trading volume. Part (2) of the corollary says that equalization of the market depths is sufficient to guarantee equalization of the expected profits of the monopolist insider in Kyle (1985) and the expected utility of the single investor in Glosten (1989) if the respective noise terms are scaled appropriately (i.e., by choosing an appropriate value of k). Notice that, by construction, the expected profits of the single investor is zero in Glosten (1989), so $E(V_g)$ measures the expected variance of the investor's profits. The magnitude of this term obviously depends on R_g, just as the equalization of the market depths depend on a specific value of R_g.

By combining the results of Corollaries 3 and 4, it becomes possible to specify conditions which simultaneously ensure that, first, market depth is equalized across all three models and, second, the expected utility of Glosten's investor is equal to the expected profits of an individual informed trader in the other two models (when m = 1). From Table 1, this condition is satisfied for the following parameter configuration: k = 1/3, t = 4/7.

Corollary 5: Suppose $\Gamma_g = \Gamma_n$ and let T be as defined in Corollary 3. If $T^2/(16-8T) = 1/3$, then (1) $\Gamma_h = \Gamma_g = \Gamma_n$ and (2) $I_h = I_n = |E(V_g)|$.

IV. Conclusion

This paper has examined the robustness of the Kyle (1985) model with respect to the noise trader assumption. Two alternative representations have been considered where the uninformed traders behaved as rational agents but the batch market mechanism for price determination was left intact. The first alternative model chosen was that of Spiegel and Subrahmanyam (1992). Here, the uninformed traders are viewed as risk-averse investors who trade to hedge their endowment of risky shares. It was shown that when the uninformed traders hedge all of their endowments ("perfect hedging") then this model and that of Kyle (1985) are equivalent in the following sense: the equilibrium values of market depth, the expected total trading volume and the expected price level are the same in both models.

Equivalence was proved both for two separate cases. First, when the number of hedgers is finite and, second, for a limit economy where the number of hedgers increased to infinity in such a way that the total amount of noise in the market remained constant. In the finite case, the uninformed traders' risk-aversion parameter has to equal a specific value which is determined by the variances of the asset value and the hedgers' endowments, the information precision and the number of hedgers and informed traders. In the limit economy, equivalence holds for any value of risk-aversion.

In the model of Glosten (1989), there is a single risk-averse investor with both endowments of information and shares of the risky asset. The interpretation is that the investor's trades are motivated both by risk-sharing possibilities as well as information. A version of this model is considered where the posted-price mechanism in the original model is replaced by a batch market mechanism. Unlike the other two models, where a risk-neutral informed trader's equilibrium trades are proportional to the market depth $1/\Gamma$, the equilibrium trades of Glosten's risk-averse investor is proportional to the sum of Γ plus a risk-premium. This fact, in turn, implies that the equilibrium value of the expected price level in Glosten (1989) is strictly lower than its corresponding value in Kyle (1985). Thus these two models can never be equivalent in the sense defined above.

However, any differences in the equilibrium values of the variables is bounded from above. Further, partial equivalence is still possible because there are conditions which separately equate the values of market depth and the expected total trading volume between the models of Kyle (1985) and Glosten (1989). In fact, an additional restriction ensures the equality of market depth across all three models. Also, when the market depths are equal, the expected utility of Glosten's investor is also equal (in magnitude) with the unconditional expected profits of an individual informed trader in the other two models.

APPENDIX

Proof of Lemma 1

Let $E(v|s^1,\ldots,s^m) = as$, where $s = \sum_{i=1}^{m} s^i$. Applying Bayes' rule,

$$a = \frac{\frac{1}{\Sigma_e}}{\frac{1}{\Sigma_v} + \frac{m}{\Sigma_e}} = \frac{t}{Q}, \text{ where } Q = 1 + t(m-1).$$

Each informed trader 'i' chooses x^i to maximize $E(\pi_n^i|s^i)$, where:

$$\pi_n^i = (v - \Gamma_n x^i - \Gamma_n \sum_{j\neq i} x^j)x^i. \tag{A1}$$

Conjecture that each informed trader's trading rule is $x^i = A_n s^i$. The first-order condition for x^i obtained by differentiating $E(\pi_n^i|s^i)$ gives the optimal A_n. Γ_n is then obtained by using the optimal A_n and the rule $\Gamma_n = \mathrm{cov}(v,y_n)/\mathrm{var}(y_n)$.

Proof of Lemma 2

$$\begin{aligned} PI_n = (\Gamma_n)^2\Sigma_{y_n} &= \frac{mt\Sigma_v Q}{(1+Q)^2} + (\Gamma_n)^2\Sigma_u \\ &= mt\Sigma_v/(1+Q). \end{aligned} \tag{A2}$$

$$\begin{aligned} C_n &= \left(\frac{ts}{Q} - \Gamma_n y_n\right)\frac{ts}{\Gamma_n(1+Q)} \\ &= \left(\frac{ts}{Q} - \frac{ts}{1+Q}\right)\frac{ts}{\Gamma_n(1+Q)} \\ &= \frac{t^2s^2}{\Gamma_n Q(1+Q)^2}. \end{aligned} \tag{A3}$$

$$\begin{aligned} I_n = E(C_n) &= \frac{mt\Sigma_v}{\Gamma_n Q(1+Q)^2} \\ &= \frac{\sqrt{mt\Sigma_v\Sigma_u}}{1+Q}. \end{aligned} \tag{A4}$$

Proof of Lemma 3

$$\pi_h^j = v(u^j + w^j) - \Gamma_h u^j \left(u^j + D \sum_{m \neq j} w^m + x_h \right) \tag{A5}$$

where x_h is the trading volume of the informed traders. Because each w^j is independent of v, $\Gamma_h x_h = \Gamma_n x_n$. So:

$$E(\pi_h^j | w^j) = -\Gamma_h (u^j)^2 \tag{A6}$$

$$\begin{aligned} Var(\pi_h^j | w^j) = \Sigma_v (w^j)^2 + (u^j)^2 \left[\Sigma_v \left(1 - \frac{mt(2+Q)}{(1+Q)^2} \right) \right. \\ \left. + (\Gamma_h D)^2 (k-1) \Sigma_w \right] + 2\Sigma_v u^j w^j \frac{(2-t)}{1+Q} . \end{aligned} \tag{A7}$$

Differentiating V_h^j with respect to u^j and then equating D with the coefficient of w^j in the resulting first-order condition yields:

$$\begin{aligned} R_h D^3 (\Gamma_h)^2 (k-1) \Sigma_w + D \left[2\Gamma_h + R_h \Sigma_v \left(1 - \frac{mt(Q+2)}{(Q+1)^2} \right) \right] \\ + R_h \Sigma_v \frac{(2-t)}{Q+1} = 0 . \end{aligned} \tag{A8}$$

It follows from (A8) that since $\Gamma_h > 0$ to satisfy the second-order condition for the informed traders, D < 0 in equilibrium. Solving for Γ_h:

$$\Gamma_h D = \frac{-\sqrt{mt\Sigma_v}}{\sqrt{k\Sigma_w}} \frac{1}{Q+1} . \tag{A9}$$

Substituting for $\Gamma_h D$ in (A8) and solving for D:

$$D = \frac{2\sqrt{mt\Sigma_v/(k\Sigma_w)} - R_h\Sigma_v(2-t)}{R_h\Sigma_v(2-t) - \dfrac{R_h\Sigma_v mt}{k(Q+1)}}. \tag{A10}$$

Since D < 0, equilibrium exists if:

$$R_h\Sigma_v(2-t) > \frac{2\sqrt{mt\Sigma_v}}{\sqrt{k\Sigma_w}}. \tag{A11}$$

The denominator of D in (A10) is always positive, so (A11) is sufficient. To show this, rewrite the denominator as:

$$\begin{aligned} \text{Denom} &= R_h\Sigma_v\left[1 - \frac{mt(Q+2)}{(Q+1)^2}\right] + \frac{(k-1)mt}{k(Q+1)^2} \\ &= \frac{R_h\Sigma_v}{(Q+1)^2}[(1-t)(4+mt) + t^2] + \frac{(k-1)mt}{k(Q+1)^2} \\ &> 0. \end{aligned}$$

In the second step, the definition $Q = 1 + t(m-1)$ has been used.

Proof of Lemma 4

$PI_h = PI_n$ follows from the result $PI_i = (\Gamma_i)^2\Sigma_{y_i}$ for i=h,n.

$$\begin{aligned} I_h &= E\left(v - \frac{ts}{1+Q}\right)\frac{ts}{\Gamma_h(1+Q)} \\ &= \frac{|D|\sqrt{mt\Sigma_v k\Sigma_w}}{1+Q} \\ &= I_n|D|\frac{\sqrt{k\Sigma_w}}{\sqrt{\Sigma_u}}. \end{aligned}$$

$$C_h = \frac{(ts)^2}{\Gamma_h Q(1+Q)^2} = C_n\frac{\Gamma_n}{\Gamma_h}.$$

Proof of Corollary 1

Suppose $\Gamma_h = \Gamma_n$. Then $C_h = C_n$ from Lemma 4 and $x_n = \frac{ts}{\Gamma_n(1+Q)} = \frac{ts}{\Gamma_h(1+Q)} = x_h$. Let Σ_{x_i} be the variance of x_i. Writing std to denote the standard deviation:

$$E(|y_n|) = \frac{std(y_n)}{\sqrt{2\pi}} = \frac{\sqrt{\Sigma_u + \Sigma_{x_n}}}{\sqrt{2\pi}} \tag{A12}$$

$$E(|y_h|) = \frac{std(y_h)}{\sqrt{2\pi}} = \frac{\sqrt{D^2 k \Sigma_w + \Sigma_{x_h}}}{\sqrt{2\pi}} \tag{A13}$$

Since $\Sigma_{x_n} = \Sigma_{x_h}$ when $\Gamma_h = \Gamma_n$, $E(|y_n|) = E(|y_h|)$ implies that $\Sigma_u = kD^2\Sigma_w$. Therefore $E(|u|) = E(|u_h|)$ by the definition of expected uninformed trading used in this paper (see footnote 8 for a discussion). Further, $I_n = I_h$ from Lemma 4.

Proof of Proposition 1

From Lemma 3, $|D| = 1$ when R_h is as given in the statement of the proposition. If, in addition, $\Sigma_u = k\Sigma_w$ then $E(|u|) = E(|u_h|)$ from the proof of Corollary 1. Also, from Lemma 4, $\Gamma_h = \Gamma_n$ and so $E(|y_n|) = E(|y_h|)$. It follows that $E(|p_n|) = \frac{\Gamma_n std(y_n)}{\sqrt{2\pi}} = \frac{\Gamma_h std(y_h)}{\sqrt{2\pi}} = E(|p_h|)$.

Proof of Proposition 2

As $k \to \infty$, $D \to -1$.

As $k \to \infty$ and $k\Sigma_w \to \Sigma_u$, $\Gamma_h \to \Gamma_n$.

Proof of Lemma 5

Let π_h be the profits of the investor. Then:

$$\pi_h = v(w + y_g) - \Gamma_g (y_g)^2.$$

$$V_h = ts(w + y_g) - \Gamma_g (y_g)^2 - (0.5) R_g (w + y_g)^2 (1-t) \Sigma_v. \tag{A14}$$

In obtaining (A14), I have used the facts that $E(v|s) = ts$ and $Var(v|s) = (1-t)\Sigma_v$.

Differentiating (A14) with respect to y_g gives:

$$y_g = \frac{ts - R_g w (1-t) \Sigma_v}{2\Gamma_g + R_g (1-t) \Sigma_v} \tag{A15}$$

and

$$\Gamma_g = \frac{R_g t (1-t) \Sigma_v}{(R_g)^2 (1-t)^2 \Sigma_v \Sigma_w - t}. \tag{A16}$$

Substituting Γ_g back into (A15) and writing $G = (R_g)^2(1-t)^2\Sigma_v\Sigma_w$ gives equation (4). Obviously, $G > t$ is necessary for $\Gamma_g > 0$.

Proofs of Corollary 2 and Proposition 3

When $m = 1$, $\Gamma_n = \frac{\sqrt{t\Sigma_v}}{2\sqrt{\Sigma_u}}$. Therefore, $\Gamma_g = \Gamma_n$ requires:

$$\sqrt{k} = \sqrt{\frac{\Sigma_u}{\Sigma_w}} = \frac{G-t}{2\sqrt{tG}}. \tag{A17}$$

Using the results of Lemma 5:

$$y_g = \frac{G-t}{G+t}\left(\frac{ts\sqrt{\Sigma_v}}{\sqrt{G\Sigma_v}} - w\right)$$

and so:

$$\mathrm{std}(y_g) = \sqrt{\frac{\Sigma_v}{G}} \cdot \frac{G-t}{\sqrt{G+t}}. \tag{A18}$$

For m = 1, $\mathrm{std}(y_n) = \sqrt{2\Sigma_u}$. Therefore, $E(|y_g|) = E(|y_n|)$ requires that:

$$\sqrt{k} = \frac{G-t}{\sqrt{2G}\sqrt{G+t}}. \tag{A19}$$

When $\Gamma_g = \Gamma_n$, $\mathrm{std}(y_g) = 2\sqrt{\Sigma_u} \cdot \frac{\sqrt{t}}{\sqrt{G+t}} = \mathrm{std}(y_n)\frac{\sqrt{2t}}{\sqrt{G+t}}$, using (A17) to get the first equality. When $\mathrm{std}(y_g) = \mathrm{std}(y_n)$, $\Gamma_g = \frac{t\sqrt{\Sigma_v}}{\sqrt{2\Sigma_u}}\frac{1}{\sqrt{G+t}} = \Gamma_n\frac{\sqrt{2t}}{\sqrt{G+t}}$, where (A19) has been used to get the first equality. Finally, $E(|p_n|) = \Gamma_n\sqrt{2\pi} = \frac{\sqrt{t\Sigma_v}}{\sqrt{2}}$ and $E(|p_g|) = \Gamma_g\sqrt{\frac{\Sigma_v}{G}}\frac{G-t}{\sqrt{G+t}} = \frac{t\sqrt{\Sigma_v}}{\sqrt{G+t}}$. Comparing, $E(|p_n|) = \frac{\sqrt{2t}}{\sqrt{G+t}}E(|p_g|)$. Clearly, equality of all these variables simultaneously occurs if $F = \frac{\sqrt{2t}}{\sqrt{G+t}} = 1$. However, from Lemma 5, $G > t$ for equilibrium to exist. Thus, $F < 1$.

Proof of Proposition 4

I must find a function $B(k) = 1-F$ with $0 < B(k) < 1$ and $B_k(k) > 0$, where B_k is the derivative of $B(k)$ with respect to k.

Write $1-F^2 = B'(k)$ and note that:

$$B(k) = \frac{B'(k)}{1+F} = \frac{B'(k)}{1+\sqrt{1-B'(k)}}. \tag{A20}$$

By definition, $1-F^2 = \frac{G-t}{G+t}$. Therefore, the function $B'(k)$ must satisfy:

$$\frac{G}{t} = \frac{1+B'(k)}{1-B'(k)}. \tag{A21}$$

Now, consider (A17) which is the condition for $\Gamma_n = \Gamma_g$. It can be viewed as a quadratic expression in the variable $\sqrt{G}$, as follows:

$$G - 2\sqrt{tk}\sqrt{G} - t = 0. \tag{A22}$$

Solving for $\sqrt{G}$ (the positive root):

$$\sqrt{G} = \sqrt{t}\,(\sqrt{k} + \sqrt{1+k}) = \sqrt{t}\,K \tag{A23}$$

where $K = \sqrt{k} + \sqrt{1+k}$. From (A21) and (A23):

$$B'(k) = \frac{K^2-1}{K^2+1}. \tag{A24}$$

Notice that $B'(k)$ is a function of k only. Further, $0 < B'(k) < 1$ and $\frac{dB'(k)}{dk} > 0$. Therefore, $B_k(k) > 0$ as well and (A20) and (A24) together defines the required function $B(k)$. $B(1) = 0.46$ and $B(8) = 0.75$, approximately.

Proof of Corollary 3

$R_g = R_s$ requires:

$$T(\sqrt{k} + \sqrt{1+k}) = 4\sqrt{k}. \tag{A25}$$

Solving for k from (A25) gives the required solution. Table 1 gives the feasible range of k and t which satisfy (A25).

Proof of Corollary 4

(1) $PI_g = [E(|p_g|)]^2 = F^2[E(|P_n|)]^2 = PI_n \cdot F^2$

(2)
$$E(v_g) = E(\pi_h) - \frac{R_g}{2} Var(\pi_h) = -\frac{R_g}{2} Var(\pi_h)$$

$$= -\frac{\sqrt{G\Sigma_v\Sigma_w}}{2} \cdot \left[(1+B_2)^2 + (B_1)^2 \frac{\Sigma_s}{\Sigma_w}\right]$$

$$= -\frac{t}{1-t} \cdot \frac{1}{2R_g}. \tag{A26}$$

For m = 1, $I_n = \frac{\sqrt{t\Sigma_v\Sigma_u}}{2}$ from Lemma 2. So $I_n = |E(V_g)| = \frac{t}{1-t}\frac{1}{2R_g}$ requires that:

$$R_g = \frac{\sqrt{t}}{1-t} \cdot \frac{1}{\sqrt{\Sigma_v\Sigma_u}}. \tag{A27}$$

From (A23), $\Gamma_g = \Gamma_n$ requires that:

$$R_g = \frac{\sqrt{t}}{1-t} \cdot \frac{1}{\sqrt{\Sigma_v\Sigma_w}} \cdot (\sqrt{k} + \sqrt{1+k}). \tag{A28}$$

If both (A27) and (A28) are to be satisfied simultaneously, $k = \frac{1}{3}$.

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Footnotes

[1]All references to the Kyle (1985) model will pertain to the static solution only and not to the continuous time model.

[2]The actual total trading volumes and price levels are not comparable since these depend on the actual volume of noise trading, which is not a choice variable in Kyle (1985). Further, the expectations are of the absolute values of these variables. See footnote 8 for further details of their calculations.

[3]In particular, the composition of the total trading volume between informed and uninformed trading is difficult to discern and so it is not required that the equilibrium values of these variables be equated as well. In models where there exists a distinct group of uninformed traders, this distinction is not important because equivalence between any two models in the sense defined here will automatically imply that the expected informed and uninformed trading volumes are also equal. In models where the same investor can trade for both informational as well as, say, for risk-sharing purposes the distinction *is* important and the expected informed and uninformed trading volumes need not be equal.

[4]Market depth is not directly observable, but is easily inferred as the ratio of the expected price level to the expected total trading volume.

[5]The version of Glosten (1989) derived in this paper is different from the original model in that I have replaced the posted-price mechanism of price determination in the original with a batch market

mechanism, keeping all the other assumptions intact. Also, only the problem of competitive market makers is studied, while the analysis of a monopolist specialist is ignored. From now on, a reference to Glosten (1989) will mean a reference to the specific version of that model derived in this paper.

[6]In a different context, Rochet and Vila (1992) have shown, for a version of Kyle (1985), that there exists a unique equilibrium independent of the distribution of uncertainty.

[7]Of course, the actual informed trading volumes will be different since market depth will be different, in general.

[8]These are actually the expectations of the absolute values of the uninformed trading volumes. Let std(r) be the standard deviation of the random variable r and $|D|$ be the absolute value of D. Then expected trading volume is $std(u)/\sqrt{2\pi}$ in the noise trading model and $\sqrt{k}|D|std(w)/\sqrt{2\pi}$ in the hedger model (see Admati and Pfleiderer (1988) for a discussion on the appropriate definition for expected trading volumes). One might expect that the expected trading volume in the hedger model should be $k|D|std(w)/\sqrt{2\pi}$. However, since noise traders in the Kyle (1985) model are a continuum, for purposes of comparing the two models it is useful to think of the k hedgers as one big hedger with trading variance $kD^2\Sigma_w$ and standard deviation of trading as given above. Notice that substituting k by $\sqrt{k}$ simply involves a monotonic transformation. The expected informed trading volumes and the expected price levels are defined in a similar way.

[9]Since informed and uninformed trading volumes are not separately defined in Glosten (1989), the only relevant variables for comparison purposes are the market depth, the expected total trading volume and the expected price levels.

Table 1: (t,k) pairs satisfying (A25) in the Appendix

t	4/7	0.63	0.64	0.65	0.66	> 0.66
k	1/3	1.22	1.78	3.02	8.01	< 0